tively) using MacCormack's explicit unsplit predictor-corrector integration scheme.

The result obtained is that the uniform flow reproduction test is automatically satisfied when the CRCLF equation is used. Using conventional forward (Δ) and backward (∇) difference operators for the predictor and corrector, respectively, it is found that the test is satisfied by the WCLF equation only if the metric derivatives are computed numerically with the same difference operator that is used to evaluate the associated flux derivative. That is,

$$(\xi_t)_\xi \simeq \frac{\Delta_i(\xi_t)}{\Delta\xi}, \quad (\xi_x)_\xi \simeq \frac{\Delta_i(\xi_x)}{\Delta\xi}, \text{ etc.} \tag{1}$$

for the predictor step, and

$$(\xi_t)_\xi^{\overline{n+1}} \simeq \frac{\nabla_i(\xi_t^{\overline{n+1}})}{\Delta\xi}, \quad (\xi_x)^{\overline{n+1}} \simeq \frac{\nabla_i(\xi_x^{\overline{n+1}})}{\Delta\xi}, \text{ etc.} \tag{2}$$

for the corrector step. Hindman notes that no constraint is placed on how the metrics ξ_t, ξ_x, etc., are to be evaluated but only how their derivatives must be evaluated. This is in distinction with the SCLF equations which Hindman shows that the metrics themselves are to be computed numerically with the same difference operator that is used to evaluate the associated flux derivative. That is,

$$x_\xi \simeq \frac{\Delta_i(x)}{\Delta\xi}, \quad y_\xi \simeq \frac{\Delta_i(y)}{\Delta\xi}, \text{ etc.} \tag{3}$$

for the predictor, and

$$x_\xi^{\overline{n+1}} \simeq \frac{\nabla_i(x)^{\overline{n+1}}}{\Delta\xi}, \quad y_\xi^{\overline{n+1}} \simeq \frac{\nabla_i(y)^{\overline{n+1}}}{\Delta\xi}, \text{ etc.} \tag{4}$$

for the corrector. In addition, the geometric conservation law (GCL) originally given by Thomas and Lombard[2] must also be integrated using MacCormack's scheme.

The implication by Hindman is that all results (or restrictions to satisfy the test) easily extend to three dimensions. This is not the case for the SCLF equation. If the results are simply extended to three dimensions, to satisfy the test we need to impose the conditions

$$\frac{\Delta_i(\xi_x/J)}{\Delta\xi} + \frac{\Delta_j(\eta_x/J)}{\Delta\eta} + \frac{\Delta_k(\zeta_x/J)}{\Delta\zeta} = 0, \text{ etc.} \tag{5}$$

for the predictor, and

$$\frac{\nabla_i(\xi_x/J)^{\overline{n+1}}}{\Delta\xi} + \frac{\nabla_j(\eta_x/J)^{\overline{n+1}}}{\Delta\eta} + \frac{\nabla_k(\zeta_x/J)^{\overline{n+1}}}{\Delta\zeta} = 0, \text{ etc.} \tag{6}$$

for the corrector. Extending results obtained by Hindman, Eq. (5), for example, would be rewritten (using transformation identities) as

$$\frac{\Delta_i(y_\eta z_\zeta - y_\zeta z_\eta)}{\Delta\xi} + \frac{\Delta_j(y_\zeta z_\xi - y_\xi z_\zeta)}{\Delta\eta} + \frac{\Delta_k(y_\xi z_\eta - y_\eta z_\xi)}{\Delta\zeta} = 0 \tag{7}$$

Now evaluating the metrics using forward difference operators it can be shown that it does not yield an identity, contrary to Hindman's two-dimensional results.

To obtain an identity, Eq. (7) should be written in the following form

$$\frac{\Delta_i[(y_\eta z)_\zeta - (y_\zeta z)_\eta]}{\Delta\xi} + \frac{\Delta_j[(y_\zeta z)_\xi - (y_\xi z)_\zeta]}{\Delta\eta} + \frac{\Delta_k[(y_\xi z)_\eta - (y_\eta z)_\xi]}{\Delta\zeta} = 0 \tag{8}$$

and similarly for the other equations. Now it is a simple matter to show that if we apply the forward difference operator to the above equation, i.e.,

$$(y_\eta z)_\zeta \simeq \frac{\Delta_k(y_\eta z)}{\Delta\zeta}, \quad (y_\zeta z)_\eta \simeq \frac{\Delta_j(y_\zeta z)}{\Delta\eta}, \text{ etc.} \tag{9}$$

and since $\Delta_i(\Delta_j A) = \Delta_j(\Delta_i A)$, then Eq. (8) is identically satisfied. The same result is obtained for all other predictor and corrector constraint equations since $\nabla_i(\nabla_j A) = \nabla_j(\nabla_i A)$ also holds. It is now noted that, contrary to Hindman's SCLF two-dimensional results, no constraint is placed on how the metrics are to be evaluated. Of course the GCL equation must still be integrated using MacCormack's scheme when the grids move.

Acknowledgment

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References

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[2] Thomas, P. D. and Lombard, C. K., "The Geometric Conservation Law—A Link Between Finite-Difference and Finite-Volume Methods of Flow Computation on Moving Grids," AIAA Paper 78-1208, July 1978.

Reply by Author to S. Paolucci

Richard G. Hindman*

Iowa State University, Ames, Iowa

Mr. Paolucci's work is essentially correct. However, Thomas and Lombard[1] recognized in 1978 that, for three-dimensional geometry, the geometric consistency required for the strong conservation law form of the governing equations is accomplished automatically by writing the conservative form of the metrics. For example,

$$\xi_x/J = \hat{i}\cdot\bar{r}_\eta \times \bar{r}_\zeta \quad (\bar{r} \text{ is the position vector})$$

is written conservatively as

$$\xi_x/J = (y_\eta z)_\zeta - (y_\zeta z)_\eta$$

Now, subsequent differencing of this relation with the same difference relations as used for the flux quantities will always yield exact geometric error cancellation provided the difference operators are linear. Therefore, Paolucci's Eq. (8)

$$\frac{\Delta_i[(y_\eta z)_\zeta - (y_\zeta z)_\eta]}{\Delta\xi} + \frac{\Delta_j[(y_\zeta z)_\xi - (y_\xi z)_\zeta]}{\Delta\eta} + \frac{\Delta_k[(y_\xi z)_\eta - (y_\eta z)_\xi]}{\Delta\zeta} = 0$$

is not new.

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*Assistant Professor in Aerospace Engineering and Computational Fluid Dynamics Institute.

Also, Paolucci states that "no constraint is placed on how the metrics are to be evaluated." In so far as the metrics x_ξ, x_η, x_ζ, y_ξ, y_η, y_ζ, z_ξ, z_η, z_ζ are concerned, this statement is true for the three-dimensional case. However, the inverse metrics ξ_x, ξ_y, ξ_z, η_x, η_y, η_z, ζ_x, ζ_y, ζ_z are most certainly constrained by the consistency requirement, even though the transformed governing equations may be formulated without the explicit appearance of these "inverse metrics."

References

[1]Thomas, P. D. and Lombard, C. K., "Geometric Conservation Law and Its Application to Flow Computations on Moving Grids," *AIAA Journal*, Vol. 17, Oct. 1979, pp. 1030-1037.

Comment on "Minimum-Weight Design of an Orthotropic Shear Panel with Fixed Flutter Speed"

Alexander H. Flax*
Institute for Defense Analyses, Arlington, Virginia

IN Ref. 1, Beiner and Librescu have presented an analysis of weight minimization for rectangular flat panels in a high supersonic flowfield subject to a panel flutter speed constraint based on aerodynamic piston theory. To simplify the problem, they have used a structural model that considers transverse shear deformation only and neglects the bending stiffness of the plate. This has the effect of reducing the linear partial differential equation for the panel flutter problem from fourth to second order. The determination of the panel flutter speed in the analysis is based on the application of Galerkin's method.

Unfortunately, as is already well-known for the problem of membrane flutter, also governed by a second-order differential equation, such reduction in the order of the differential equation may lead to difficulties in the convergence of Galerkin's method. In fact, for the two-dimensional membrane panel flutter problem with supersonic aerodynamic forces respresented by piston theory, Galerkin's method leads to finite flutter speeds for any finite number of Galerkin weighting functions, whereas, as shown by a number of authors,[2,3,4] the exact solution of the differential equation (which is, of course, itself approximate) shows the panel to be stable for all supersonic flight velocities high enough for piston theory to apply.

An examination of Beiner and Librescu's Eq. (1) shows that, like the equation for membrane flutter, it must lead to exact solutions for frequency which are real under all supersonic flight conditions to which piston theory applies. Therefore, panel flutter speeds derived by Galerkin's method are spurious. The general demonstration of this property of the differential equation (i.e., that it can have no other than real eigenvalues) follows from the proof that for ordinary linear differential equations with variable real coefficients any second-order equation may be put in self-adjoint form by multiplying it by the non-vanishing function

$$I(x)=\frac{1}{a_0}\exp\int_{x_0}^{x}\frac{a_1(\rho)}{a_0(\rho)}\mathrm{d}\rho$$

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*President. Honorary Fellow AIAA.

where x is the independent variable, $a_0(x)$ is the coefficient of the second derivative term, and $a_1(x)$ is the coefficient of the first derivative term. In addition, the boundary conditions must satisfy the conditions for self-adjointness of the differential system.[5] Since in the present case of a partial differential equation the term in the lateral dimension is already in the self-adjoint form, the reduction in the streamwise direction, which has both a first and a second derivative term, may be carried out exactly as in the case of an ordinary differential equation, rendering the entire equation self-adjoint.

The self-adjoint properties of the partial differential equation for panel flutter when only transverse shear deformations are accounted for can easily be seen for the special case of constant panel thickness. For this case, Eq. (1) of Ref. 1 becomes

$$\bar{G}_{13}\frac{\partial^2 w}{\partial y^2}+\bar{G}_{23}\frac{\partial^2 w}{\partial x^2}-\lambda\frac{\partial w}{\partial x}+\omega^2\mu w=0 \tag{1}$$

where, as in the original paper, $\bar{G}_{13}$ and $\bar{G}_{23}$ are, respectively, the elastic moduli in transverse shear in the lateral and longitudinal directions nondimensionalized with respect to a convenient shear modulus, G_{ref}, and direction 3 corresponds to the direction transverse to the plate. $\lambda=\kappa p_\infty M_\infty/G_{\mathrm{ref}}h$, where κ is the polytropic gas coefficient, and p_∞ and M_∞ are the pressure and Mach number, respectively, in the undisturbed flow. $\mu=\rho/G_{\mathrm{ref}}$ where ρ is the mass density of the plate. The motion in time is taken to be given by $\bar{w}=we^{i\omega t}$, where ω is the circular frequency.

If b is the transverse dimension of the plate, Eq. (1) has a solution of the form

$$w=\frac{\sin m\pi y}{b}F(x)$$

where m is any integer, so that

$$\bar{G}_{23}\frac{\mathrm{d}^2F}{\mathrm{d}x^2}-\lambda\frac{\mathrm{d}F}{\mathrm{d}x}+\left(\omega^2\mu-\frac{m^2\pi^2}{b^2}\bar{G}_{13}\right)F=0 \tag{2}$$

This may be further reduced by substituting

$$F(x)=f(x)\exp\frac{\lambda}{2\bar{G}_{23}}x$$

to give

$$\bar{G}_{23}\frac{\mathrm{d}^2f}{\mathrm{d}x^2}+\left(\omega^2\mu-\frac{m^2\pi^2}{b^2}\bar{G}_{13}-\frac{\lambda^2}{4\bar{G}_{23}}\right)f=0 \tag{3}$$

Finally the solution of this obviously self-adjoint equation is given by

$$f=\sin(n\pi x/a)$$

where a is the dimension of the panel in the x direction and n may be any integer. Thus

$$\omega^2=\frac{1}{\mu}\left[\frac{n^2\pi^2}{a^2}\bar{G}_{23}+\frac{m^2\pi^2}{b^2}\bar{G}_{13}+\frac{\lambda^2}{4\bar{G}_{23}}\right] \tag{4}$$

The frequencies of the panel are seen to be all real and all increase monotonically with the aeroelastic Mach number parameter, λ. No panel flutter is possible for any value of λ.

Bolotin[4] has discussed the convergence (or lack thereof) of the Galerkin method for the membrane of infinite aspect ratio in terms of the properties of the infinite determinant generated by Galerkin's method with an infinite number of weighting functions. He shows that the infinite determinant does not converge for the membrane but that with finite plate bending stiffness (i.e., a fourth order differential equation) the determinant converges although ever more slowly as the